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Electronic Properties of the Vortex State in Cuprate Superconductors

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The superconducting state of the cuprates in the presence of a magnetic field has been investigated very actively in the past few years through measurements of electrical and thermal transport, ac conductivity, specific heat, and other quantities. The observed behavior is not well understood; it probes the nature of quasiparticles, vortices, and their interactions in a superconductor with nodes in the pair amplitude. We summarize here experimental results and our attempts to understand the phenomena.

1. INTRODUCTION

The discovery of superconductivity in the cuprates (1) has led to an intense exploration of this fascinating family of systems. It has become clear that their normal state (i.e., the state above T_c) is unique and is qualitatively unlike known metals (2, 3). The nature of this metallic state and the mechanism of superconductivity are active open questions, 12 years and more than 100,000 papers later. Meanwhile, due to the efforts of a large number of solid state chemists and physicists, excellent, well-characterized single crystalline and thin film samples have become available, and we now have wide variety of high quality experimental results on these systems. The systems are experimentally over-determined!

In the superconducting state, the excitation gap $\Delta_{\mathbf{k}}$, whose magnitude measures the energy required to break up a pair $(\mathbf{k}\sigma, -\mathbf{k}-\sigma)$ and create quasiparticles, appears to be a strong function of wavevector $\mathbf{k}$ (4–6). It has nodes and changes sign as $\mathbf{k}$ moves over the Fermi surface; a form that fits the data well is $\Delta_k = \Delta_0(\cos k_x a - \cos k_y a)$ where a is the lattice constant. Clearly the gap vanishes at the points $k_x = \pm k_y$, and there are low energy quasiparticle excitations around these nodal points. This $d_{x^2-y^2}$ order leads to characteristic, novel temperature dependences for physical properties (6–8). Since the gap vanishes at $k_x = \pm k_y$, external perturbations such as magnetic fields can have strong and even singular effects on the excitation spectrum. For example, it has been argued on the basis of an electronic thermal conductivity kink observed at low fields and temparatures (9) that there is a new fully gapped phase in this region of T and H. In this article, I first summarize our experimentally derived picture of the superconducting state (Section 2). Some features of the vortex or mixed state, e.g., specific heat, tunnelling density of states, ac conductivity, and thermal transport, are then mentioned (Section 3). Attempts (10–12) to describe theoretically a possible new fully gapped complex order parameter phase in the presence of vortices are summarized (Section 4). Because of the anisotropy of the superconductor, the energy of the vortex lattice depends on its orientation with respect to the nodal line. A calculation of this effect (13, 14) and the consequent transition in the vortex lattice structure are then outlined (Section 5). The basic new features affecting electronic properties of the superconducting phase are the unusual, gapless Dirac excitation spectrum of the quasiparticles and the nonlocality of the order parameter $\Delta(\mathbf{r}, \mathbf{r}')$. Because of the latter, any variation of the order parameter with position (i.e., with the electron pair center of mass coordinate $\mathbf{R} = \{(\mathbf{r} + \mathbf{r}')/2\}$ affects the internal state of the pair (described by the $\mathbf{k}$ dependence of $\Delta_{\mathbf{k}}$; $\mathbf{k}$ is the Fourier transform of the internal or relative coordinate $(\mathbf{r} - \mathbf{r}')$). This coupling has been obtained (15) and is briefly mentioned below (Section 6). It has a number of qualitative consequences. Finally, (Section 7) I discuss electronic longitudinal and transverse or Hall-like thermal conductivities (16–18) which have an unusual temperature and field dependence. The concluding section (Section 8) mentions some open questions.

2. THE SUPERCONDUCTING STATE

There is now considerable experimental evidence supporting the view that the superconducting state in the cuprates is a coherent superposition of spin singlet electron pairs with the electrons in a relative d orbital state (5). For example, from angle resolved photoemission (ARPES) experiments it is clear that the energy of the highest occupied quasiparticle state in a Bi2212 superconductor depends on its in-plane momentum $\mathbf{k}_{||}(=k_x, k_y)$ (4). For optimally doped Bi-2212 with $T_c \simeq 90$ K the minimum energy has the

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form $|\Delta_{\mathbf{k}}| = \Delta_0|\cos(k_x a) - \cos(k_y a)|$ with $\Delta_0 = 25$ meV. Whether $\Delta_{\mathbf{k}}$ actually has a zero or not is uncertain to within about 10 meV, the energy resolution of the ARPES measurements. A broadly similar dependence of $\Delta_{\mathbf{k}}$ with $\mathbf{k}$ is observed irrespective of doping and of the specific chemical system. The ARPES measurements also reveal a Fermi surface in the (k_x, k_y) plane with a large area approximately equal to $(1 - x)$ electrons per unit cell where x is the hole fraction. The Fermi surface shape seems "universal" as well. Another important observation from photoemission is that the quasiparticles near the Fermi surface are well defined; a quasiparticle $\mathbf{k}$ has a well-defined energy $E_{\mathbf{k}}$ to within instrumental resolution. By contrast, in the normal state (above T_c), the spectral density is rather broad and asymmetric. Quasiparticles are not well-defined excitations in the normal state, for reasons that are not clear.

ARPES experiments are sensitive to the magnitude of $\Delta_{\mathbf{k}}$ and not its sign or phase. A number of phase sensitive experiments using atomically engineered Josephson junctions show that the order parameter does have positive and negative parts depending on orientation with respect to the crystal axes. They thus suggest that $\Delta_{\mathbf{k}} \simeq \Delta_0\,[\cos(k_x a) - \cos(k_y a)]$, at least in optimally doped superconductors. Such a gap function changes sign across the lines $k_x = \pm k_y$.

Indirect experimental evidence for this kind of gap is provided, for example, by the temperature dependence of the superfluid density, as measured by the penetration depth. The superfluid density decreases linearly with temperature. This is most naturally understood as being due to thermally excited quasiparticles, whose density is proportional to temperature T for a gap that grows linearly around the nodal points (zeroes) (6–7). The measured rate of decrease of the superfluid density with temperature is in reasonable agreement with what is expected from the gap function determined by ARPES (19), though there are systematic deviations.

A superconducting gap of the form $\Delta_k = \Delta_0[\cos(k_x a) - \cos(k_y a)]$ is expected in a lattice model if nearest neighbor electrons form a spin singlet pair and if the pair amplitude along the y-axis has a sign opposite to that of the amplitude along the x-axis ("d-wave" pairing). If electrons repel each other strongly when on the same site and have an antiferromagnetic Heisenberg nearest neighbor spin exchange coupling J, nearest neighbor pairing is energetically favored. However, it is not entirely clear as to why pair amplitudes along x- and along y-axes are out of phase. There are several measurements suggesting a (relatively small) admixture of same sign (or extended s) pairing (20). In the normal state, there is evidence for a precursor pseudogap with $d_{x^2-y^2}$ symmetry.

The simplest realistic picture of the superconducting state in cuprates is thus one of a gap corresponding to nearest neighbor d-wave pairing and well-defined planar quasiparticle excitations consistent with this. The layers are Josephson coupled with their nearest neighbors. By contrast, there is no general agreement on how to describe the normal state. A number of consequences of this model for the superconductor have been explored for many physical properties, with mixed success. One area where understanding remains poor is the effect of an external magnetic field H ($\ll H_{c_2}$, the upper critical field). In the next section, some experimental results will be summarized; attempts to understand these will be the subject of the bulk of this article.

3. CUPRATE SUPERCONDUCTOR IN A MAGNETIC FIELD

It was established very early that cuprates are extreme type II superconductors (with penetration depth $\lambda \simeq 1500$ Å and coherence length $\xi \simeq 15$ Å so that the Ginzburg–Landau parameter $(\lambda/\xi) \sim 100 \gg 1$). It is also well known that a magnetic field enters in units of a flux quantum $\phi_0 = (hc/2e)$, thus establishing electron pairs as basic entities. The arrangement of vortices (ordered lattice, glass or fluid) as a function of field, temperature, and disorder is a major subject which I do not touch upon, except in Section 5, where the question of the vortex lattice structure at $T = 0$ and in the limit of no disorder (pinning) is discussed. Vortex motion in an applied electric field leads to longitudinal and Hall resistivities that differ qualitatively from that observed in conventional superconductors. One widely studied phenomenon is the anomalous sign of the Hall effect (21), which seems to be correlated with doping level and cannot be reconciled with expectations based on standard Ginzburg–Landau theory. We do not discuss these phenomena here, but concentrate on vortices at rest; it might be argued that if these are not well understood, effects dependent on vortex motion are even harder to make sense of.

Four kinds of experimental results are mentioned here: specific heat, tunneling spectroscopy, microwave conductivity, and thermal transport, all in the vortex state.

(i) Specific Heat

The electronic specific heat in a magnetic field has been measured extensively in $YBa_2Cu_3O_{7-\delta}$ (22, 23). One motivation is the prediction of Volovik (24) that the superfluid velocity arround a vortex adds to that of the quasiparticle and thus induces a density of states at the nodes proportional to $\sqrt{B}$. Consequently one has a linear in T specific heat with this field dependence. Careful measurements of the specific heat (23) indicate the following. There is a linear in T term at $B = 0$, of order 0.4 mJ/g at. K. Its origin is unknown. There is a field dependent term typically 20 to 10% of this. It does not have the simple $TB^{1/2}$ dependence predicted; this could be because the above form is expected for large fields and low temperatures, i.e., for $x = [(T/T_c)(B/B_{c_2})^{1/2}] \ll 1$. The measurements are in the regime $0.3 < x < 5$. The specific heat approximately fits a scaling plot $(C_{\text{vortex}}/\gamma_n T) = k(B/B_{c2})^{1/2}F(x)$, the measured functional dependence being consistent with limiting forms of $F(x)$ for small and large x. However, due to the existence of a sizeable zero field linear specific heat, and in the absence of any systematic theory for the density of quasiparticle states due to a dense collection of vortices at $T \neq 0$ in the presence of other disorder, the interpretation of the magnetic field dependence is tentative.

(ii) Bound States in the Vortex

Tunnelling spectroscopy of flux lines in $YBa_2Cu_3O_{7-\delta}$ reveals (25) a peak in the density of states when the STM tip is in the vortex core region. It strongly suggests a bound state confined to the core with a binding energy of about 5.5 meV. Such a bound state is not expected for a d-wave superconductor, since the order parameter vanishes in some directions unlike an s-wave superconductor where the isotropic but radial distance dependent pair potential of the core can confine quasiparticles. Some calculations suggest (26) no bound state, while others (27) do. However, these approaches do not include nonlocal effects correctly, as we shall see below (15). Thus the theoretical question of whether there are vortex core bound states in a d-wave superconductor cannot be considered settled. It is interesting to note, however, that in BSCCO, which is much more two dimensional, no such bound state peaks were seen (28). This could be because of possible vortex motion in the latter smearing out such a peak or because the bound state energy is higher than the gap (25). Further investigation is clearly needed.

(iii) Microwave Conductivity

The microwave conductivity of thin films of BSCCO shows an unusual magnetic field dependence. Mallozzi *et al.* (29) find that σ_2 the imaginary part of the conductivity has a part decreasing as $\sqrt{B}$, in the range $B < 5T$. The term is nearly temperature independent. They argue that this cannot be attributed to vortex dynamics, but that it is due to a magnetic field dependent reduction in the superfluid density, since $\sigma_2(\omega) = -(n_s e^2/mi\omega)$ in an equilibrium superconductor where n_s is the superfluid density. Again, this unusual field dependence is attributed to excitation of circulating supercurrent by a vortex and consequent "paramagnetic" reduction in superfluid stiffness. However, the $\sqrt{B}$ dependence is expected according to semiclassical arguments (29,7) at $T = 0$, or more precisely, for $x = (T/T_c)/(B/B_{c_2})^{1/2} \ll 1$ as mentioned above. The measurements are mostly in the opposite range. Thus while the experimental results are interesting, the theoretical explanation is not conclusive.

(iv) Thermal Transport

Some of the most interesing insights into the electronic properties of the vortex state are from measurements of thermal conductivity. The electronic thermal conductivity (if it can be separated from the lattice or phonon contribution) is a probe of energy current relaxation of quasiparticles interacting with *static* vortices. (In the absence of an external electric field, vortices do not move). Here, perhaps the most unexpeted result is the sharp thermal conductivity anomaly reported by Krishana *et al.* (9).

Krishana *et al.* (9) find in very clean single crystals of BSSCO that the electronic thermal conductivity (in the *ab*-plane, for B perpendicular to it) decreases as a function of the magnetic field, but suddenly flattens out at a sharply defined B value B^* (Fig. 1). This critical field, typically of order a few Tesla, depends on temperature T^* approximately as $\sqrt{B^*}$. Krishana *et al.* argue that while the decrease in κ_T with B is understandable as a consequence of scattering by a larger number of vortices, the sudden field independence or vanishing of κ_T implies that there are no thermally excited quasiparticles beyond B^* to carry the heat current. This can happen if a fully gapped phase is stable beyond B^*.

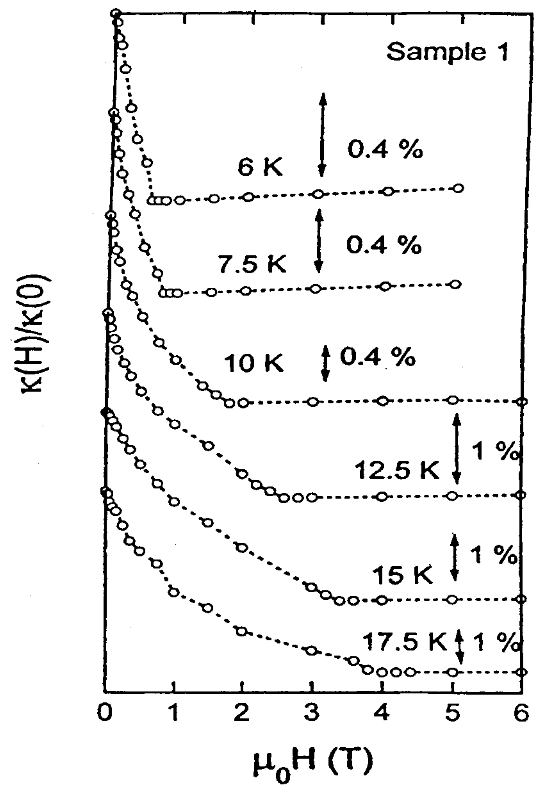

FIG. 1. Thermal conductivity of a clean single superconducting crystal of Bi-2212, with magnetic field $\mu_0 H$ perpendicular to the *ab*-plane, for various values of $\mu_0 H$ in Tesla, at different temperatures indicated. (From Ref. (9)).

Thus the phase diagram of a cuprate superconductor has a new fully gapped phase inside it at low T and B (Fig. 2). Subsequent experiments (30) find a great deal of hysteresis associated with this transition (considerable difference of behavior in field cooled and zero field cooled samples); often the "transition" is smooth and is best described as a crossover (31). Thus the existence of a true phase transition at low T and B, and its nature, are both not fully settled yet. I discuss below a calculation (11) which shows that an $Ad_{x^2-y^2} + iBd_{xy}$ order parameter is inevitable in a magnetic field. In such a case, the gap does not vanish. This order parameter is a consequence of quasiparticle vortex interaction which produces phase shifted Andreev scattering, thus acting as a source of out of phase pair potential. Other explanations involve an analogy with the quantum Hall effect (10) and the Landau-like quantization of quasiparticle energy levels in a magnetic field (32). The possibility of a new phase in the presence of a magnetic field in a $d_{x^2-y^2}$ superconductor is a very interesting issue.

In general, the thermal conductivity κ_{xx} shows a peak as a function of temperature T at $B = 0$ (17, 18, 31). The peak occurs around and below T_c. Since there is other evidence, e.g., from ARPES, that quasiparticles are ill defined above T_c and well defined below T_c, it seems natural that they do not transport heat current above T_c but do so below T_c, thus leading to a peak in $\kappa_{xx}(T)$. The phonon contribution decreases smoothly with T across T_c. On applying a magnetic field the thermal conductivity is observed to decrease (Fig. 3). This could be because of extra scattering of quasiparticles by vortices, which increases with their density, i.e., with B. There is considerable experimental evidence (9, 17, 18, 31) that the phonon thermal conductivity is independent of B, i.e., the phonon vortex scattering is weak. Thus $\kappa_{xx}(B, T)$ probes a consequence of the quasiparticle vortex interaction. A phenomenological fit to the data has been obtained by Ong and co-workers (17). They find that

$$\kappa_{xx}(H, T) = \frac{\kappa_e^0(T)}{1 + p(T)\ |H|} + \kappa_B(T), \qquad [1]$$

where the mean free path $l(T)$ associated with $\kappa_e^0(T)$, and $p(T)$ have the same temperature dependence. Both increase dramatically as temperature decreases; $l(T)$ grows approximately as T^{-2} from about 40 Å near T_c ($\simeq$ 83 K) to about 3000 Å near 10 K. The simple field dependence of Eq. [1] is not understood. Even more interestingly, the Hall thermal conductivity $\kappa_{xy}(H, T)$ has a linear H dependence for small H (9, 33), the slope of which is *also* related to $l(T)$. More precisely, writing the Hall angle θ_Q as $\theta_Q = \kappa_{xy}/\kappa_{xx}$, and

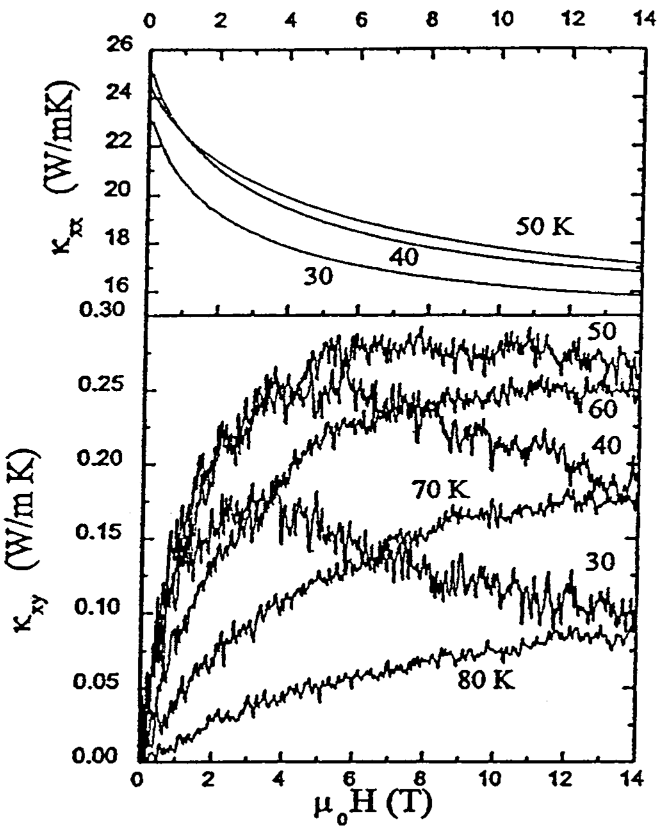

FIG. 3. The diagonal thermal conductivity k_{xx} in untwinned $YBa_2Cu_3O_7$ as a function of $\mu_0 H$ in Tesla, for different temperatures (upper panel) and the Hall thermal conductivity (lower panel). (From Ref. (33))

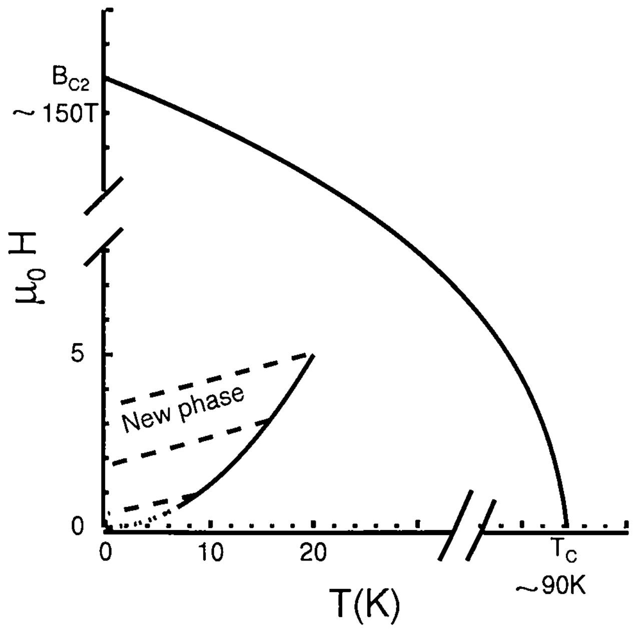

FIG. 2. Possible phase diagram of a cuprate superconductor in a magnetic field. The mixed or vortex or Abrikosov phase for $\mu_0 H < H_{c_2} \simeq 150\ T$ and $T < T_c \sim 90$ K is indicated. At low temperatures and fields, a possible new fully gapped phase (shown) is suggested by thermal conductivity anomalies.

expressing θ_Q as

$$\theta_Q = (\ell_H eB/\hbar k_F), \quad [2]$$

it appears (33) that the Hall length ℓ_H has the same temperature dependence as $l(T)$! On plotting the *electrical* Hall angle (θ_e/B), one finds that it follows the same temperature dependence and has the same size, namely (θ_e/B) and (θ_Q/B) fall on the same curve (33). It is not clear if this is a coincidence or reflects the persistence of certain kinds of fluctuations above and below T_c.

At very low temperatures ($T < 0.2$ K) it appears that $\kappa_{xx}(H, T)$ *increases* with increasing H (34). The increase has been fitted to a $\sqrt{H}$ form whose coefficient decreases with increasing Zn doping. This dependence has been connected with the magnetic field induced density of states near the node (24,7).

We thus see that there is a rich variety of novel effects observed for d-wave superconductors in a magnetic field. They are not well understood. In what follows, I outline some of our efforts to understand and predict specific phenomena, as well as to develop general underlying ideas.

4. id_{xy} ORDER INDUCED BY VORTICES

I discuss here a specific mechanism (11) due to the coupling between quasiparticles and the supercurrent as well as order parameter inhomogeneity associated with vortex, which necessarily causes out of phase d_{xy} order in a $d_{x^2-y^2}$ superconductor. The effect is nonlocal in space, i.e., it arises from gradient terms.

Vortices affect the pair potential in two ways. The potential acquires a phase ϕ which changes by 2π on going round a vortex. The magnitude of the order parameter goes to zero at the core of the vortex and rises to its equilibrium value over a distance of order ξ, the pair size. Because of the former, there is a nonzero superfluid momentum $m\mathbf{v}_s(\mathbf{r}) = [(\hbar/2)\nabla\phi - (e\mathbf{A}/c)]$ which adds to the electron momentum. The quasiparticle Hamiltonian can be written as

$$H = \frac{1}{2m}\sum_\sigma \int d\mathbf{r}\psi_\sigma^+(\mathbf{r})\{\mathbf{p} + m\mathbf{v}_s(\mathbf{r})\}^2\psi_\sigma(\mathbf{r}) + \int d\mathbf{r}\, d\mathbf{r}' [\Delta_m(\mathbf{r}-\mathbf{r}', \{\mathbf{R}-\mathbf{R}_\ell\})\psi_\uparrow^+(\mathbf{r})\psi_\downarrow^+(\mathbf{r}') + hc]. \quad [3]$$

Here $\mathbf{R}$ is the center of mass coordinate $\{(\mathbf{r}+\mathbf{r}')/2\}$, and $\mathbf{R}_\ell$'s are locations of the vortices. In the momentum representation, and the Nambu formalism for pair states, Eq. [3] can be expressed as

$$H = H_0 + (H_\theta + H_m + H_{KE}), \quad [4a]$$

where

$$H_0 = \sum_k a_k^+(\tilde{\varepsilon}_k\tau_3 + \Delta_k\tau_1)\, a_k \quad [4b]$$

$$H_\theta = \sum_{k,q} a_k^+\{\hbar\mathbf{k}\cdot\mathbf{v}_s(\mathbf{q})\}\, a_{k-q} \quad [4c]$$

$$H_m = -\Delta_0\sum_{k,q} f_{kq}(a_k^+\tau_1 a_{k-q}) \quad [4d]$$

and H_{KE} is the kinetic energy of the superfluid. The first term, H_o, in Eq. [4b] describes a uniform $d_{x^2-y^2}$ superconductor (if Δ_k is given by $\Delta_k = \Delta_0(\cos k_x a - \cos k_y a)$). The terms H_θ, H_m, and H_{KE} represent effects due to vortices. H_θ corresponds to the scattering of a quasiparticle by the supercurrent around a vortex. The term H_m describes the change in the pair potential due to the vortex core; f_{kq} is a form factor of this potential. The last term in Eq. [4a], H_{KE}, is the superfluid kinetic energy $(mv_s(r)^2/2)$.

The question of interest is the effect of the terms of Eqs. [4c] and [4d] on the pair potential. To find this, we treat these terms as perturbations and calculate out of phase pair correlations $\langle a_k^+\tau_2 a_k\rangle$ induced by them. Figure 4 shows that there is a term in this pair amplitude of first order in both H_0 and H_m. The physical meaning of this process is obvious: an electron of momentum $\mathbf{k}$ gets scattered to another state $(\mathbf{k}-\mathbf{q})$ by the superfluid flow; this shifts its phase. It is then Andreev reflected to a hole state by the pair potential inhomogeneity associated with the vortex core. Detailed calculation shows (11) that this term has an ik_xk_y

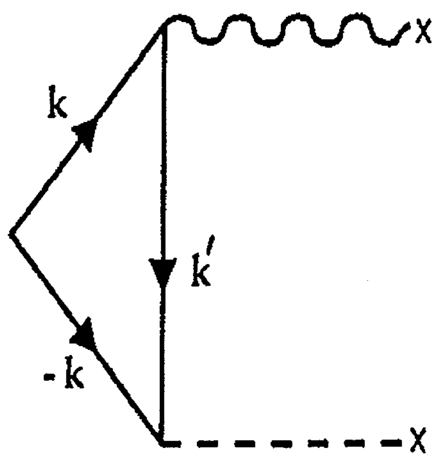

FIG. 4. A process leading to an id_{xy} pair amplitude involving scattering of a quasiparticle $\mathbf{k}$ to a state $\mathbf{k}' = (\mathbf{k}-\mathbf{q})$ by the supercurrent around the vortex and Andreev reflection by local pair potential inhomogeneity near the vortex core.

dependence on **k**, namely

$$\langle a_k^+ \tau_2 a_k \rangle = \sum_q \{i(\mathbf{k} \times \mathbf{q}) \cdot \hat{e}_z\}\{\varepsilon_{\mathbf{k}-\mathbf{q}}\} g_{kq} \qquad [5a]$$

$$\simeq ik_x k_y \sum_q (q_x^2 - q_y^2) g_{kq}. \qquad [5b]$$

In Eq. [5a], the first term is from the $\mathbf{k} \cdot \mathbf{v}_s(\mathbf{q})$ coupling, and the second term is from the intermediate quasiparticle state. Equation [5b] makes it clear that the k dependence is of the form $ik_x k_y$ and the contribution is from nonlocal (of order q^2) anisotropic effects $(\overline{q_x^2 - q_y^2} \neq 0)$. g_{kq} is a symbol for all the remaining factors.

Other processes involving H_θ also induce an id_{xy} order. It can also be shown that, dynamically, a gap function with this symmetry is generated. Thus a fully gapped state is inevitable at zero temperature, in a magnetic field, due to quasiparticle vortex interaction. The size of this term is estimated to be about $\Delta_{xy} \sim 10(n_v/n)\Delta_0$ where n_v is the vortex density and n is the electron density. At nonzero temperatures, entropic effects disfavor a fully gapped phase so that a transition to a $d_{x^2-y^2}$ state occurs at some $T^* \sim \Delta_{xy}$. This yields numerically the right temperature scale, but a temperature dependence linear in B.

The above discussion assumes no particular arrangement of vortices. It is clear, however, that a lattice of voritces can produce a Bragg/superconductive gap. We (12) have calculated the electronic structure of a vortex lattice using the Bogoliubov–de Gennes equations. We find that there is a gap in the excitation spectrum (at $T = 0$) at the nodal points. The gap is actually of order $\langle p \cdot \mathbf{v}_s(\mathbf{G}) \rangle \sim (k_F G/m)$. Now the reciprocal lattice vector $G \propto \sqrt{B}$, so that the magnetic field dependence is close to that observed (9). Calculating the pair amplitude $\langle u_k v_k \rangle$, we find it complex, with an imaginary $k_x k_y$ component. There are some problems with the slow convergence of the Hamiltonian matrix in Brillouin zone space largely due to the slow decrease $(1/G)$ of $v_s(G)$ with G. These are being overcome by partitioning and perturbative evaluation of large G effects. Thus, there seems to be considerable theoretical evidence for a vortex induced id_{xy} phase, which leads to sizeable gaps of the right B dependence for the lattice.

5. GROUND STATES OF THE VORTEX LATTICE

In an s-wave superconductor, vortices repel each other isotropically: the repulsive vortex–vortex interaction is screened at distances of order the penetration depth λ. The vortex lines are also straight. Under these conditions, the stable arrangement is a close packed triangular lattice with vortices as far away from each other as possible. This is the well-known Abrikosov lattice. In a $d_{x^2-y^2}$ superconductor, the interaction between vortices is anisotropic. The effect of this anisotropy on the stable vortex lattice structure is an obvious question of interest. Experimentally, there is evidence from small angle neutron scattering (35) and from scanning tunnelling spectroscopy (25) in YBCO systems that the vortices are arranged in a distorted, centered rectangle. The order seems short-ranged, however. Part of the unit cell distortion is due to anisotropy in the penetration depths λ_a and λ_b. This leads only to a distorted triangular lattice irrespective of the vortex density. We are interested in anisotropic effects beyond these.

There have been calculations of the consequences of order parameter anisotropy on vortex lattice structure near T_c (36). These predict a distorted centered rectangle structure. Recently, Franz *et al.* (13) considered the problem of vortex interaction deep in the $d_{x^2-y^2}$ phase, at $T = 0$, using a semilocal approximation and found that as a result of the dependence of this interaction on the angle with respect to the nodal directions, there is a continuous deformation of the vortex lattice unit cell with vortex density. We (14) have looked at this problem without using further approximations and find that there is a sudden transition at about 5 T from a triangular lattice structure to a centered square, the reciprocal lattice vectors of both structures being oriented along the nodal direction. I discuss the physical idea and the calculations below.

Two vortices interact via the currents or current fluctuations they produce in the quasiparticle fluid. The appropriate susceptibility for this is the current correlation function $\chi_{\alpha\beta}(\mathbf{q})$ where α and β are Cartesian components of the electron momentum, and $\mathbf{q}$ is the momentum transfer. For a vortex lattice, $\mathbf{q}$s relevant are the reciprocal lattice vectors $\mathbf{G}$. The energy of pairwise interaction of the vortices is

$$\Delta_E = \left(\frac{m^2}{2}\right) \sum_{\mathbf{G}} \{v_s(\mathbf{G})\} \chi_{\alpha\beta}(\mathbf{G}) \{\mathbf{v}_s(-\mathbf{G})\}_\beta \qquad [6]$$

We calculate this with the susceptibility appropriate to a superconductor described by the Hamiltonian Eq. [4b]. As noted Franz *et al.* (13), the susceptibility is nonanalytic and anisotropic in q, there being a linear term of the form $|\max(g_x, g_y)|$. We find, however, that relative structural stability is determined by the anisotropy in the quadratic term [in $\chi_{\alpha\beta}(q)$]. This requires careful treatment of the nonlocal contribution from k dependence of Δ_k. We consider the ground state energy as a function of two angles describing the vortex lattice: one describes the shape of the unit cell (a centered rectangle) and the other its orientation with respect to the $k_x = k_y$ line. We find that at low fields, the ground state is a triangular lattice, while at high fileds it is a centered square with $\mathbf{G}$ along the nodal line. The transition occurs at about 5 T, the field being determined by the size of the anisotropy. At low fields, the nodal orientation effect is small and the Abrikosov structure is stable. At higher fields, the orientational effect is maximized by an orientationally commensurate square structure. The dependence of the transition field on parameters like Δ_0 and the Fermi surface shape is being investigated. At $T \neq 0$, there are entropic effects not inlcluded in the theory which affect the structural transition boundary. There is as yet no experimental test of the above prediction.

6. NONLOCAL EFFECTS IN A *d* WAVE SUPERCONDUCTOR

The new effects mentioned above arise from the fact that the pair potential has zeroes and varies with pair momentum $\mathbf{k}$. I outline here a more general description of the consequences of a nonlocal order parameter (15).

Consider, for example, the equations for the single particle Greens functions. These Gor'kov equations for $G(\mathbf{r}\tau, \mathbf{r}'\tau')$ describe how an electron at $\mathbf{r}\tau$ moves to a point $\mathbf{r}'\tau'$ because of its own kinetic energy and because of the pair potential $\Delta(\mathbf{r}, \mathbf{r}')$. Now when vortices are present, the latter necessarily has a phase which changes by 2π on going round a vortex ($\mathbf{R} = \mathbf{R}_\ell$). This phase can be transferred to each electron by a gauge transformation and adds to its momentum by an amount $m\mathbf{v}_s(\mathbf{R})$ as discussed earlier. This line of thinking suggests that the natural variables in which to discuss the equations of motion are $((\mathbf{r} + \mathbf{r}')/2) = \mathbf{R}$ and $(\mathbf{r} - \mathbf{r}') = \boldsymbol{\rho}$. The former is the center of mass coordinate in which quanatities vary smoothly, on a scale ξ or greater, and the latter is the relative or internal coordinate of the pair, which has a typical range a (lattice constant) of k_F^{-1}. The two degrees of freedom are connected when the order parameter or pair potential is nonlocal e.g., $\Delta(\mathbf{r} - \mathbf{r}') = 0$ for $\mathbf{r} = \mathbf{r}'$ and nonzero only for $\mathbf{r} - \mathbf{r}'$ being nearest neighbors. The center of mass motion (dependence of $\Delta(\mathbf{r}, \mathbf{r}')$ on $\mathbf{R}$) affects internal motion to the first order in $(1/k_F\xi)$. This is a semiclassical correction to the local approximation of Gor'kov equations. Since $(1/k_F\xi)$ is not very small in cuprates (it is of order 1/5 or so), such corrections can be sizeable.

The mixed representation $(\mathbf{k}, \mathbf{R})$, where $\mathbf{k}$ is the Fourier transform of $\boldsymbol{\rho} = (\mathbf{r} - \mathbf{r}')$, is convenient for describing the equations of motion. We find (15) that the equation for $G(\mathbf{k}, \mathbf{R}; \omega)$ has the form

$$[\omega - \tilde{\varepsilon}_k - (\hbar\mathbf{k}/m)\cdot\{(\mathbf{P}_R/2) + m\mathbf{v}_s(\mathbf{R})\}]G(\mathbf{k}, \mathbf{R}; \omega) - \Delta(\mathbf{k}; \mathbf{R})\, F^+(\mathbf{k}, \mathbf{R}; \omega) - (i/2)\{(\partial\Delta(\mathbf{k}, \mathbf{R})/\partial k_\alpha)(\partial F^+(\mathbf{k}, \mathbf{R}; \omega)/\partial R_\alpha) - (k \rightleftharpoons R)\} = 1. \quad [7]$$

In Eq. [7], ω is the frequency and the third term in the square bracket on the left hand side and the last term on that side are all $(1/k_F\xi)$ corrections. If they are neglected, one has a local version of the equation determining Green's function. The Volovik approximation amounts to keeping only the $\mathbf{k}.\mathrm{v}_s(\mathbf{R})$ term. This neglects the quantum fluctuation term $(\hbar\mathbf{k}\cdot\mathbf{P}_R/2m)$ due to center of mass motion, and the last term is novel, being nonzero only when $\Delta_\mathbf{k}$ depends on $\mathbf{k}$. We see that this term couples the internal state $\mathbf{k}$ to center of mass motion $\mathbf{R}$. If the anomalous Green's function F depends on the center of mass coordinate as near a vortex or an interface, the internal state $\Delta_\mathbf{k}$ changes. For example, if $\Delta_\mathbf{k} \sim (\cos k_x a - \cos k_y a)$, a term going as $i(\partial\Delta_k/\partial k_x)$ or $i\sin k_x a$ is induced. This is the source of the ik_xk_y order induced by a vortex, for example. The consequences for a vortex are being worked out, e.g., in the Eilenberger or semiclassical approximation (37).

For a single vortex, in a region of space where the order parameter changes rapidly, i.e., near the core, so that $(\partial F/\partial R_\alpha)$ is the largest, sizeable id_{xy} order is induced. Thus the local order parameter near the vortex core is complex and therefore nonzero; it can sustain a bound state, such as is observed (25).

7. THERMAL TRANSPORT IN THE VORTEX STATE

At temperatures and fields such that the vortices are not ordered, quasiparticles scatter off the dense collection of vortices at rest. We (38) have investigated the consequences of this scattering, superimposed on other sources of quasiparticle lifetime, in the Born approximation. We calculate the conductivity. Since the vortices have well-developed local short range order, the scattering from them is modified by the structure factor $S(\mathbf{q})$ (39) which depends on positional correlations between vortices. $S(\mathbf{q})$ has a strong peak at $|\mathbf{q}| \simeq |\mathbf{G}_s|$ where $\mathbf{G}_s$ is the smallest reciprocal lattice vector of the crystalline solid formed at lower T but at the same vortex density. Thus, scattering with wavevector transfer $\mathbf{G}_s$ is picked out, in contrast to the absence of such a preference in a random arrangement vortex gas. Further, since $|\mathbf{G}_s|$ is small in relation to the Fermi, wavevector k_F, the quasiparticle scattering is by small angles, and the scattering in term ($\cos\theta$ term in current relaxation rate) is large. All this leads to a quasiparticle relaxation rate by vortices proportional to their density (38). This adds to other relaxation processes, as suggested by the phenomenological fit Eq. [1].

There is a transverse scattering of quasiparticles by a vortex which directly contributes to the Hall thermal resistance (38). This leads, at low fields, to the observed behavior.

At very low temperatures, there are indeed very few quasiparticle states near the node, and the scattering from vortices causes states in this region, as from any other random scatterer (40). Thus there are field-induced states at low T that carry the energy current (41). At high temperatures, there are thermally excited quasiparticle states present, and as discussed above, random scattering by vortices leads to decreasing thermal conductivity as the density of vortices increases. Thus both kinds of experimental observations can be reconciled in a single framework.

8. CONCLUSION AND OUTLOOK

I have described above some of the unusual electronic properties of the vortex phase of high T_c cuprate superconductors and our attempts to understand them in terms of nonlocal effects peculiar to superconductors with a linearly vanishing gap at nodal points. A number of questions remain unanswered. Perhaps the most interesting from a basic point of view is the origin of the quasiparticle relaxation process that continues smoothly through T_c (33) from above it to well below it. What are these fluctuations? Why are they different for transverse and longitudinal processes above T_c and become identical (?) below T_c? Another issue of interest is the nonmonotonic behavior of the Hall thermal resistivity (Fig. 4) (16, 33). Why does it peak at relatively low fields of order a few Tesla, with a field scale that decreases with temperature? To this one must add perhaps the most significant unexplained new phenomenon in the superconducting state, namely the origin of the sharply defined 41 meV inelastic neutron scattering peak at $\mathbf{Q} = (\pi, \pi, \pi)$ (42).

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